# *Spiral plat sans courbes terminales*

## *Poids du spiral et anisochronisme en position verticale*

### *Cas d'une montre bracelet*

### Caractéristiques du spiral *dextre*

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Dimensions $ép = 0.03\,mm$ $ha = 0.15\,mm$ $S = 4.5 \times 10^{-3}\,mm^2$ $TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm$ $d1_{sp} = 1.1\,mm$ $p_{sp} = 0.135\,mm$ $n_{sp} = 12.667$

$L := L_{sp}$ $L = 11.182\,cm$ $\psi_0 := 2\cdot\pi\cdot n_{sp}$ $\psi_0 = 4.56 \times 10^3\,deg$

Positions du piton $r_P := 0.5\cdot d2_{sp}$ $\alpha_P := 0$ $x_P := r_P\cdot\cos(\alpha_P)$ $y_P := r_P\cdot\sin(\alpha_P)$

$x_P = 2.26\,mm$ $y_P = 0\,mm$

Position du point d'attache à la virole $r_V := 0.5\cdot d1_{sp}$ $\alpha_V(\theta) := \psi_0 + \theta$ $x_V(\theta) := r_V\cdot\cos(\alpha_V(\theta))$ $y_V(\theta) := r_V\cdot\sin(\alpha_V(\theta))$

$z_P := x_P + \mathrm{i}\cdot y_P$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi}$ $r_s(\alpha) := r_P - a\cdot\alpha$ $x_{0s}(\alpha) := r_s(\alpha)\cdot\cos(\alpha)$ $y_{0s}(\alpha) := r_s(\alpha)\cdot\sin(\alpha)$ $z_0(\alpha) := r_s(\alpha)\cdot\exp(\mathrm{i}\cdot\alpha)$

$r'(\alpha) := \dfrac{d}{d\alpha}r_s(\alpha)$ $s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_P^2 - r_s(\alpha)^2\right)$ $s(\alpha) := r_P\cdot\alpha - \dfrac{a}{2}\cdot\alpha^2$ $s(\psi_0) = 11.182\,cm$

Amplitude stationnaire du balancier $\theta_0 = 270\,deg$

### Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

### Première approximation de la déformée du spiral

$$z'_0(\alpha) := \left[-a + \mathrm{i}\cdot(r_P - a\cdot\alpha)\right]\cdot\exp(\mathrm{i}\cdot\alpha)$$

$$z_1(\theta,\alpha) := z_P + \int_0^{\alpha}\left[-a + \mathrm{i}\cdot(r_P - a\cdot\alpha')\right]\cdot\exp\left[\mathrm{i}\cdot\alpha'\cdot\left[1 + \frac{\theta}{L}\cdot\left(r_P - \frac{a}{2}\cdot\alpha'\right)\right]\right]d\alpha'$$

### Première approximation du déplacement du centre de gravité

$$\zeta_{1s}(\theta) := \frac{1}{L}\cdot\int_0^{\psi_0} z_1(\theta,\alpha)\cdot r_s(\alpha)\,d\alpha \qquad \zeta_{1s}(\theta_0) = 0.199 + 0.037\mathrm{i}\,mm$$

Déplacement de la virole libre

$$\mathbf{\Delta_1}(\theta) := \frac{\mathrm{i}\cdot\theta}{L}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot\exp\left(\mathrm{i}\cdot\theta\cdot\frac{s(\alpha)}{L}\right)\cdot r_s(\alpha)\,d\alpha \qquad u_1(\theta) := Re(\mathbf{\Delta_1}(\theta)) \qquad v_1(\theta) := Im(\mathbf{\Delta_1}(\theta))$$

$\xi_{1s}(\theta) := \frac{d}{d\theta}v_1(\theta) - u_1(\theta) \qquad \eta_{1s}(\theta) := \frac{d}{d\theta}u_1(\theta) - v_1(\theta) \qquad \xi_{1s}(\theta_0) = 0.199\,mm \qquad \eta_{1s}(\theta_0) = 0.037\,mm$

## Calcul du déplacement du centre de gravité

$x_1(\theta,\alpha) := Re(z_1(\theta,\alpha)) \qquad y_1(\theta,\alpha) := Im(z_1(\theta,\alpha))$

$$\sigma 2_1(\theta) := \frac{1}{L}\cdot\int_0^{\psi 0} z_1(\theta,\alpha)\cdot\overline{z_1(\theta,\alpha)}\cdot r_s(\alpha)\,d\alpha \qquad \frac{\sigma 2_1(\theta_0)}{2} = 1.188\,mm^2$$

$$q2_{1s}(\theta) := \frac{1}{L}\cdot\int_0^{\psi 0} y_1(\theta,\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad p2_{1s}(\theta) := \frac{1}{L}\cdot\int_0^{\psi 0} x_1(\theta,\alpha)^2\cdot r_s(\alpha)\,d\alpha$$

$$k_{1s}(\theta) := \frac{1}{L}\cdot\int_0^{\psi 0} x_1(\theta,\alpha)\cdot y_1(\theta,\alpha)\cdot r_s(\alpha)\,d\alpha$$

$q2_{1s}(\theta_0) = 1.168\,mm^2 \qquad p2_{1s}(\theta_0) = 1.207\,mm^2 \qquad k_{1s}(\theta_0) = 0.027\,mm^2$

$$\Sigma 2_1(\theta) := \frac{1}{L^2}\cdot\int_0^{\psi 0} s(\alpha)\cdot z_1(\theta,\alpha)\cdot\overline{z_1(\theta,\alpha)}\cdot r_s(\alpha)\,d\alpha \qquad \frac{\Sigma 2_1(\theta_0)}{2} = 0.429\,mm^2$$

$$Q2_{1s}(\theta) := \frac{1}{L^2}\cdot\int_0^{\psi 0} s(\alpha)\cdot y_1(\theta,\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad P2_{1s}(\theta) := \frac{1}{L^2}\cdot\int_0^{\psi 0} s(\alpha)\cdot x_1(\theta,\alpha)^2\cdot r_s(\alpha)\,d\alpha$$

$Q2_{1s}(\theta_0) = 0.42\,mm^2 \qquad P2_{1s}(\theta_0) = 0.439\,mm^2$

$$\xi_{2s}(\theta) := \frac{d}{d\theta}v_1(\theta) - \frac{\Sigma 2_1(\theta)}{\sigma 2_1(\theta)}\cdot u_1(\theta) - \frac{v_1(\theta)}{2\cdot\sigma 2_1(\theta)}\cdot\frac{d}{d\theta}\sigma 2_1(\theta) \qquad \xi_{2s}(\theta_0) = 0.067\,mm$$

$$\eta_{2s}(\theta) := \frac{d}{d\theta}u_1(\theta) - \frac{\Sigma 2_1(\theta)}{\sigma 2_1(\theta)}\cdot v_1(\theta) + \frac{u_1(\theta)}{2\cdot\sigma 2_1(\theta)}\cdot\frac{d}{d\theta}\sigma 2_1(\theta) \qquad \eta_{2s}(\theta_0) = 0.046\,mm$$

$$\sigma 2 := \frac{1}{L}\cdot\int_0^{\psi 0} z_0(\alpha)\cdot\overline{z_0(\alpha)}\cdot r_s(\alpha)\,d\alpha \qquad \sigma 2 := \frac{r_P{}^2 + r_V{}^2}{2} \qquad \sigma 2 = 2.705\,mm^2$$

$$\kappa := \frac{1}{\sigma 2\cdot L^2}\cdot\int_0^{\psi 0} s(\alpha)\cdot z_0(\alpha)\cdot\overline{z_0(\alpha)}\cdot r_s(\alpha)\,d\alpha \qquad \kappa := \frac{1}{3}\cdot\frac{r_P{}^2 + 2\cdot r_V{}^2}{r_P{}^2 + r_V{}^2} \qquad \kappa = 0.352$$

$\xi_s(\theta) := \frac{d}{d\theta}v_1(\theta) - \kappa\cdot u_1(\theta) \qquad \eta_s(\theta) := \frac{d}{d\theta}u_1(\theta) - \kappa\cdot v_1(\theta) \qquad \xi_s(\theta_0) = 0.064\,mm$

$\eta_s(\theta_0) = 0.044\,mm$

$$\zeta(\theta) := \frac{1}{L}\cdot\int_0^{\psi 0} z_0(\alpha)\cdot e^{i\cdot\theta\cdot\frac{s(\alpha)}{L}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s(\alpha)}{L} - \kappa\right)\right]\cdot r_s(\alpha)\,d\alpha \qquad \zeta(\theta_0) = 0.064 + 0.044i\,mm$$

## Approximations de Haag

$$\xi_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} x_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha \qquad \eta_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha \qquad \sigma 2 := \frac{1}{2}\cdot\left(r_P^{\,2} + r_V^{\,2}\right)$$

$$q2_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} y_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad p2_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad k_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha$$

$$q2_{0s} = 1.352\,mm^2 \qquad q2_{0s} - \eta_{0s}^{\,2} = 1.35\,mm^2 \qquad p2_{0s} = 1.353\,mm^2 \qquad p2_{0s} - \xi_{0s}^{\,2} = 1.353\,mm^2$$

$$\sigma 2 = 2.705\,mm^2$$

### Formule de Haag

$$\boldsymbol{OP} := r_P \qquad \boldsymbol{OV} := r_V\cdot e^{\mathrm{i}\cdot\psi_0}$$

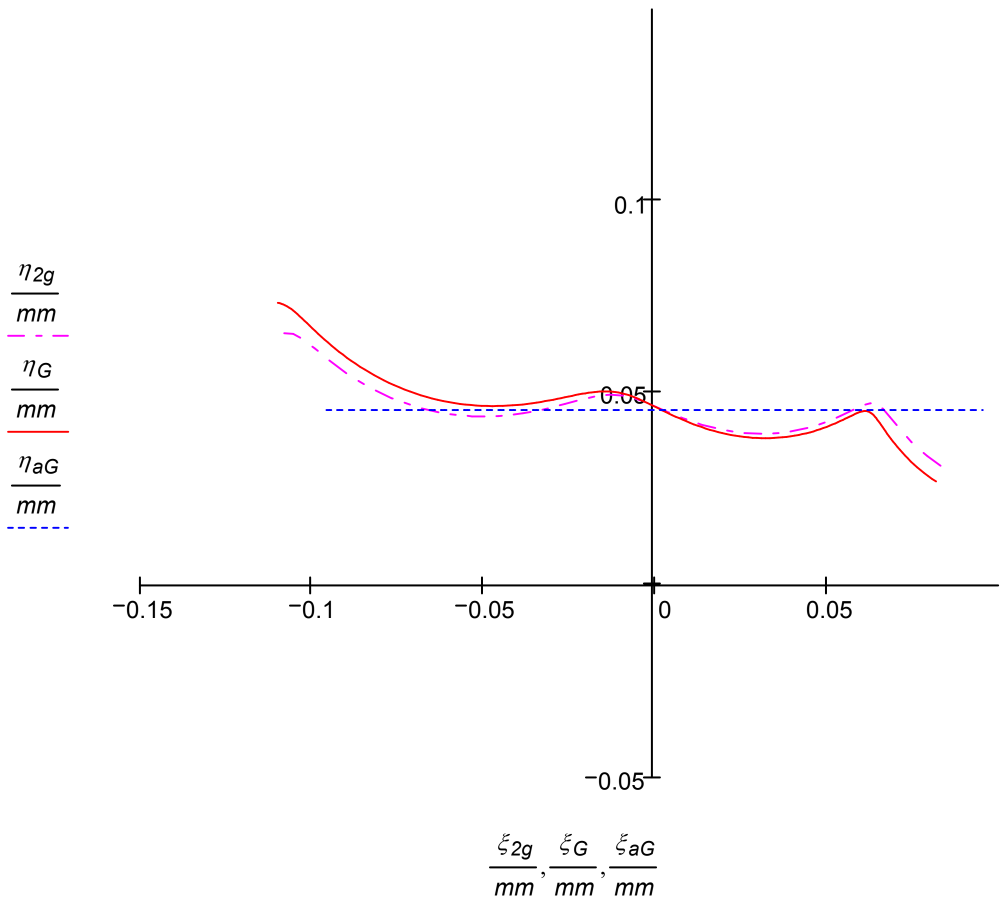
$$\zeta_{ah}(\theta) := \frac{r_P}{L}\cdot\left(\mathrm{i} + \frac{\theta}{3}\right)\cdot\boldsymbol{OP} \qquad \zeta_{ah}(\theta_0) = 0.072 + 0.046\mathrm{i}\,mm$$

## Graphes du déplacement du centre de gravité

$$n := 201 \qquad i := 0 .. n-1 \qquad \Delta\theta := \frac{4\cdot\pi}{n-1} \qquad \theta_i := -2\cdot\pi + i\cdot\Delta\theta$$

$$m := 41 \qquad j := 0 .. m-1 \qquad \Delta\theta_m := \frac{4\cdot\pi}{m-1} \qquad \theta_{m_j} := -2\cdot\pi + j\cdot\Delta\theta_m \qquad \xi_{2g_j} := \xi_{2s}\left(\theta_{m_j}\right) \qquad \eta_{2g_j} := \eta_{2s}\left(\theta_{m_j}\right)$$

$$\xi_{G_i} := Re\left(\zeta\left(\theta_i\right)\right) \qquad \eta_{G_i} := Im\left(\zeta\left(\theta_i\right)\right) \qquad \xi_{aG_i} := Re\left(\zeta_{ah}\left(\theta_i\right)\right) \qquad \eta_{aG_i} := Im\left(\zeta_{ah}\left(\theta_i\right)\right) \qquad \xi_{2g_i} := 0 \qquad \eta_{2g_i} := 0$$



## Perturbation de période - spiral non déformé en position de repos

### Calcul par intégrations numérqiues

$$\eta(\theta) := Im(\zeta(\theta)) \qquad Gamma(\theta) := -m_s \cdot g \cdot \frac{d}{d\theta}\eta(\theta)$$

$$\theta(\varphi) := \theta_0 \cdot cos(\varphi) \qquad \Delta(\theta_0) := \frac{L}{2\cdot\pi\cdot\theta_0\cdot E\cdot I_{33}} \cdot \int_0^{2\cdot\pi} \Gamma(\theta_0 \cdot cos(\varphi)) \cdot cos(\varphi)\, d\varphi \qquad \Delta(\theta_0) = -2.64 \times 10^{-5}$$

$$\zeta(\theta) := \frac{d}{d\theta}\zeta(\theta) \qquad \zeta(\theta_0) = 5.254 \times 10^{-3} - 5.89i \times 10^{-3}\, mm$$

$$\zeta(\theta) := \frac{1}{L} \cdot \int_0^{\psi_0} z_0(\alpha) \cdot exp\left(i \cdot \theta \cdot \frac{s(\alpha)}{L}\right) \cdot \left[i \cdot \left(2 \cdot \frac{s(\alpha)}{L} - \kappa\right) - \theta \cdot \frac{s(\alpha)}{L} \cdot \left(\frac{s(\alpha)}{L} - \kappa\right)\right] \cdot r_s(\alpha)\, d\alpha$$

$$f(\theta_0, \alpha) := \int_0^{2\cdot\pi} exp\left(i \cdot \theta_0 \cdot \frac{s(\alpha)}{L} \cdot cos(\varphi)\right) \cdot \left[i \cdot \left(2 \cdot \frac{s(\alpha)}{L} - \kappa\right) - \theta_0 \cdot \frac{s(\alpha)}{L} \cdot \left(\frac{s(\alpha)}{L} - \kappa\right) \cdot cos(\varphi)\right] \cdot cos(\varphi)\, d\varphi$$

$$f(\theta_0, \alpha) := 2\cdot\pi\cdot\left(\kappa - 2\cdot\frac{s(\alpha)}{L}\right) \cdot J1\left(\theta_0 \cdot \frac{s(\alpha)}{L}\right) + \pi \cdot \left(\kappa - \frac{s(\alpha)}{L}\right) \cdot \theta_0 \cdot \frac{\alpha}{\psi_0} \cdot \left(J0\left(\theta_0 \cdot \frac{s(\alpha)}{L}\right) - Jn\left(2, \theta_0 \cdot \frac{s(\alpha)}{L}\right)\right)$$

$$f(\theta_0, \alpha) := 2\cdot\pi\cdot\frac{s(\alpha)}{L} \cdot \left[\left(\kappa - \frac{s(\alpha)}{L}\right) \cdot \theta_0 \cdot J0\left(\theta_0 \cdot \frac{s(\alpha)}{L}\right) - J1\left(\theta_0 \cdot \frac{s(\alpha)}{L}\right)\right]$$

$$Z(\theta_0) := \frac{1}{2\cdot\pi\cdot\theta_0\cdot L} \cdot \int_0^{\psi_0} z_0(\alpha) \cdot f(\theta_0, \alpha) \cdot r_s(\alpha)\, d\alpha \qquad Z(\theta_0) = -8.404 \times 10^{-4} + 4.304i \times 10^{-4}\, mm$$

$$Z(\theta_0) := \frac{1}{L^2} \cdot \int_0^{\psi_0} z_0(\alpha) \cdot s(\alpha) \cdot \left[\left(\kappa - \frac{s(\alpha)}{L}\right) \cdot J0\left(\theta_0 \cdot \frac{s(\alpha)}{L}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s(\alpha)}{L}\right)\right] \cdot r_s(\alpha)\, d\alpha$$

$$Delta(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z(\theta_0)) \qquad Delta(\theta_0) = -2.64 \times 10^{-5}$$

$$\mu(\theta_0) := -86400 \cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = 2.281} \qquad \boxed{\mu(180 \cdot deg) = 1.172}$$

### Approximation par développement en série

$$f_B(\theta_0) := (\kappa - 1) \cdot J0(\theta_0) - \frac{1}{\theta_0} \cdot J1(\theta_0) \qquad f_{1B}(\theta_0) := \frac{1}{L} \cdot \left[-2 \cdot J0(\theta_0) + (\kappa - 1) \cdot (J0(\theta_0) - \theta_0 \cdot J1(\theta_0))\right]$$

$$Z_a(\theta_0) := -r_P{}^2 \cdot \frac{\kappa}{L^2} \cdot \mathbf{OP} + \frac{1}{L} \cdot \left[-(i \cdot r_V + 2 \cdot a) \cdot f_B(\theta_0) + r_V{}^2 \cdot f_{1B}(\theta_0)\right] \cdot \mathbf{OV}$$

$$\delta_a(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_a(\theta_0)) \qquad \delta_a(\theta_0) = -2.196 \times 10^{-5}$$

$$\mu_a(\theta_0) := -86400 \cdot \delta_a(\theta_0) \qquad \boxed{\mu_a(\theta_0) = 1.898} \qquad \boxed{\mu_a(180 \cdot deg) = 0.783}$$

## Approximations de Haag

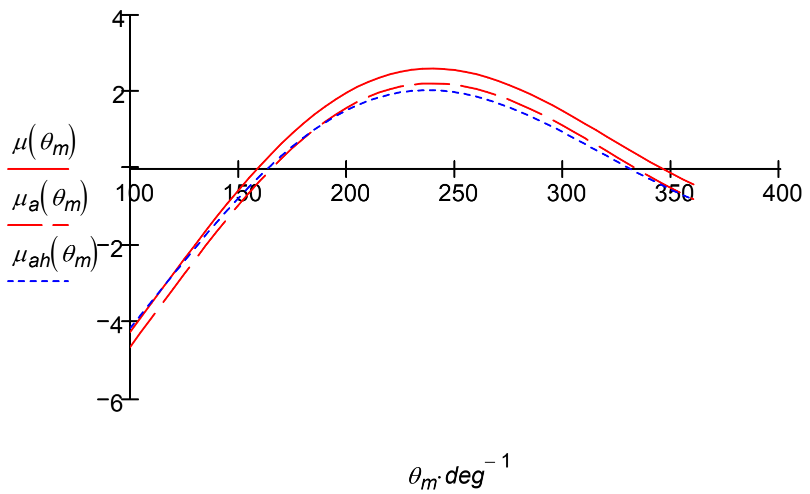
$z_{V0} := x_V(0) + i \cdot y_V(0) \qquad z_{V0} = -0.275 - 0.476i\,mm$

$U(\theta_0) := \frac{2}{3} \cdot J0(\theta_0) + \frac{1}{\theta_0} \cdot J1(\theta_0) \qquad Z_{ah}(\theta_0) := \frac{i \cdot r_V}{L} \cdot U(\theta_0) \cdot \mathbf{OV}$

$\delta_{ah}(\theta_0) := -g \cdot \frac{m_s}{E \cdot I_{33}} \cdot r_V^{\,2} \cdot cos(\alpha_V(0)) \cdot U(\theta_0)$

$\mu_{ah}(\theta_0) := -86400 \cdot \delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = 1.699} \qquad \boxed{\mu_{ah}(180 \cdot deg) = 0.804}$

$\theta_m := 100 \cdot deg, 105 \cdot deg .. 360 \cdot deg$




$\theta_1 := 170 \cdot deg \qquad \theta_1 := racine(U(\theta_1), \theta_1) \qquad \theta_1 = 163.44\,deg$

$\theta_2 := 320 \cdot deg \qquad \theta_2 := racine(U(\theta_2), \theta_2) \qquad \theta_2 = 330.542\,deg$